Fast Terminal Sliding Mode Control of an Electromechanical System

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Abstract
In this paper, a fast terminal sliding mode controller is designed for speed control of a DC brushed servo motor system. The fast terminal sliding mode controller was developed after the sliding mode control and has been used frequently in the scientific world. The aim of this study is to see in practice why it is needed when there are other methods, why it is more preferred, what problems it provides solutions to, and its deficiencies, if any. For this purpose, it was compared with both its ancestor sliding mode control and basic terminal sliding mode control, which is the slow type of the method.
The fast terminal sliding mode control approach improves the convergence of the system states on the sliding manifold in finite-time. In this study, the stability of the system is justified by employing the Lyapunov stability theorem. Simulations on the DC motor system show that the fast terminal sliding mode control method provides better performance compared to the conventional sliding mode control and the basic terminal sliding mode control approaches. As a result of the study, it was seen that the method in question met the promised performance and speed criteria.
Keywords: Sliding mode control, Fast terminal sliding mode, Electromechanical systems, Finite-time convergence, Stability.

1. INTRODUCTION
Research efforts on designing new control approaches in the control engineering field are expanding.In this context, the comprehensive comparisons between different types of control methods fill an important gap in the research area [1,2]. In this paper, the performance of the fast terminal sliding mode control (FTSMC) method which has an increasing popularity in recent years is compared with the sliding mode control (SMC) approach and the basic terminal sliding mode control (TSMC) approach. A DC motor is employed as the electromechanical testbed for this purpose.
DC motors are used in numerous different plants in the industryby reason of their various decisive advantages such as high efficiency, fast response, and simple control features [3,4]. Based on these advantages, several different control approaches are used to control DC motors such as neural network-based control [5,6], fuzzy control [7,8], and SMC [9-11].
The SMCmethodis a straightforwardnonlinear control approachthatis resistant to uncertainties and disturbances. However, because of the linear sliding surfaces employed in the conventional SMC the state convergence time of the system could be infinite. To address that problem a new type of SMC was proposed in the early 1990s called TSMC [12,13]. The TSMC approach ensuresthe convergence of the system states in finite-timehowever convergence would be slow. Using an additional linear term in the sliding surface establishes the FTSMC approach which can overcome this problem. The FTSMC method guarantees fast convergence [12,14].
In this study, a fast terminal sliding mode controller is designed for speed control of a DC motor. The performance of the designed controller is set side by side with the conventional SMC and the basic TSMC methods.The rest of this paper isstructured as follows. The dynamic model of the DC motorispresented in section 2. Section 3 introduces the basics of the conventional SMC method. Section 4 contains anoverview of the basic TSMC and FTSMC approaches. The simulation results are provided in Section 5 and the conclusion of the paper is presented in Section 6.

2. DC MOTOR MODEL

A motor can be defined as an electromechanical device that generates a motion for voltage input. In other words, it is a mechanical result produced by an electrical input [15].
The schematic of the DC motor is shown in Figure 1.

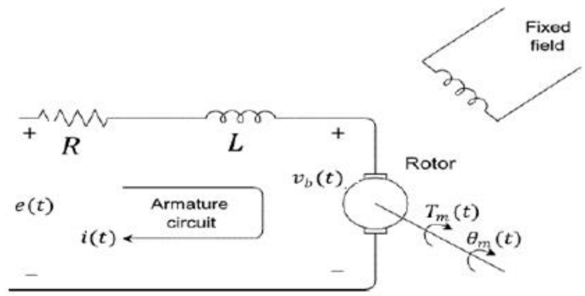

Figure 1: DC motor schematic

DC motor produces torque. The torque (τ) can be defined as:

$$\tau = K_t i(t) \tag{1}$$

where K_t is the torque constant and i(t) is the motor current.
Similarly, the induced electromotive force can be defined as:

$$V_b = K_b \dot{\theta}(t) \tag{2}$$

whereV_b is electromotive force; K_b is electromotive force constant which relates physical characteristics of the motor and $\dot{\theta}(t)$ is angular shaft speed.
According to newton's dynamics of the DC motor can be written:

$$J\,\ddot{\theta}(t) = \sum \tau_i = -\,d\,\dot{\theta}(t) + K_t i(t) \tag{3}$$

where J is the constant of the moment of inertiaand d is the viscous friction constant.
Also, the electrical model of the DC motor can be written:

$$u(t) - V_b = L\,\frac{d\,i(t)}{dt} + R\,i(t) \tag{4}$$

where u(t) is the control signal; R is resistance and L is inductance. Substituting Eq. (2) into Eq. (4), the following equation is obtained:

$$u(t) = L\,\frac{d\,i(t)}{dt} + R\,i(t) + K_b \dot{\theta}(t) \tag{5}$$

Using Eqs. (3-5), there will be obtained two fundamental differential equations as follows:

$$\ddot{\theta}(t) = -\frac{d}{J}\dot{\theta}(t) + \frac{K_t}{J} i(t) \tag{6}$$

$$\frac{d\,i(t)}{dt} = -\frac{R}{L} i(t) - \frac{K_b}{L}\dot{\theta}(t) + \frac{1}{L}u(t) \tag{7}$$

The state space model of the DC motor can be written:

$$\frac{d}{dt}\begin{bmatrix}\dot{\theta}(t)\\ i(t)\end{bmatrix}=\begin{bmatrix}-\frac{d}{J} & \frac{K_t}{J}\\ -\frac{K_b}{L} & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}\dot{\theta}(t)\\ i(t)\end{bmatrix}+\begin{bmatrix}0\\ \frac{1}{L}\end{bmatrix}u(t) \tag{8}$$

If $\omega(s)$ is written instead of $\dot{\theta}$in Eqs. (7) and (8) taking Laplace transform, the following equations are obtained:

$$s\,\omega(s)=-\frac{d}{J}\omega(s)+\frac{K_t}{J}i(s) \tag{9}$$

$$i(s)=\frac{K_b}{s\,L+R}\omega(s)+\frac{1}{s\,L+R}u(s) \tag{10}$$

SubstitutingEq. (10) into Eq. (9) yields

$$s^2\omega(s)=-\left[\frac{R}{L}+\frac{d}{J}\right]s\,\omega(s)-\frac{d\,R+K_t\,K_b}{L\,J}\omega(s)+\frac{K_t}{L\,J}u(s) \tag{11}$$

First, the inverse Laplace transform is performed. Secondly, there is an additional gain K_{amp} offered by amplifiers. Then, the engine angular speed is converted from "rad/s" to rpm:

$$\ddot{\omega}(t)=-\left[\frac{R}{L}+\frac{d}{J}\right]\omega(t)-\frac{d\,R+K_tK_b}{L\,J}\omega(t)+\frac{K_{amp}T_{rpm}K_t}{L\,J}u(t) \tag{12}$$

where $\omega(t)$ is angular shaft speed; $u(t)$ is the control signal; K_{amp}(and equals 9.6) is an additional gain. When the motor angular speed is converted rad/s to rpm: T_{rpm} (equals $60/2\pi$) is obtained, and also$d(x,t)$ is the sum of uncertainties and distortions [16].
If $x\ [x_1, x_2]^T=[\omega, \dot{\omega}]^T$is the state vector, the next nonlinear dynamic equations are found:

$$\dot{x}_1(t)=x_2(t)\quad \dot{x}_2(t)=f(x,t)+g(x,t)\,u(t)+d(x,t) \tag{13}$$

When Eq. (12) and Eq. (13) are combined:

$$f(x,t)=-\left[\frac{R}{L}+\frac{d}{J}\right]x_2(t)-\frac{d\,R+K_t\,K_b}{L\,J}x_1(t) \tag{14}$$

$$g(x,t)=\frac{K_{amp}\,K_t\,T_{rpm}}{L\,J} \tag{15}$$

where $g(x,t)$ is a positive constant;$d(x,t)\le d_{max}$; d_{max} is a positive unknown coefficient [17].
The parameters of the servo motor model are given in Table 1.

Table 1 Parameters of the DC Motor [18]

Parameter	**Value**
K_b	0.052 Vs / rad
K_t	0.052 Nm/ A
L	0.0025 H
R	2.5 W

d	0.000425 Nms/rad
J	0.0001218 $kg\ m^2/s^2$

3. SLIDING MODE CONTROL

A sliding mode controller design procedure consists of two main stages: forcing the system states to a predefined reference position and then holding them at that position, which is the sliding mode phase.
One must design a control signal u(t) to force the system states to the predefined reference position. To design the control input for the system introduced in Eq. (13), the tracking error e(t) can be defined as:

$$e(t) = x_1(t) - r(t) \tag{16}$$

where r(t) is the reference signal. The first and the second derivatives of the tracking error e(t) in Eq. (16) with respect to the time can be written as:

$$\dot{e}(t) = \dot{x}_1(t) - \dot{r}_1(t) \tag{17}$$

$$\ddot{e}(t) = \dot{x}_2(t) - \ddot{r}(t) \tag{18}$$

Also, a sliding surface s(t) can be written as:

$$s(t) = \lambda e(t) + \dot{e}(t) \tag{19}$$

where λ is a positive constant.
Taking the derivative of s(t) with respect to the time yields:

$$\dot{s}(t) = \lambda \dot{e}(t) + \dot{x}_2(t) - \ddot{r}_2 \tag{20}$$

Substituting $\dot{x}_2$ introduced in Eq. (13) into Eq. (20) yields:

$$\dot{s}(t) = \lambda \dot{e}(t) + f(x,t) + g(x,t)\, u(t) + d(x,t) - \ddot{r}_2 \tag{21}$$

The control signal u(t) can be defined as:

$$u(t) = u_{eq}(t) + u_{sw}(t) \tag{22}$$

where $u_{eq}(t)$ is the equivalent control signal; $u_{sw}(t)$ is the switching signal.
To ensure that the state trajectory stays on the sliding manifold, the equivalent control $u_{eq}(t)$ can be chosen as:

$$u_{eq}(t) = -\frac{1}{g(x,t)} \left[\lambda \dot{e}(t) + f(x,t) - \ddot{r}_2 \right] \tag{23}$$

If the Lyapunov candidate function is chosen as:

$$V(t) = \frac{1}{2} s^2(t) \tag{24}$$

Differentiating the Lyapunov function and rearranging it using the system equations introduced in Eq. (13) yields:

$$\dot{V}(t)= s(t)\big[\lambda\dot{e}(t) + f(x,t) + g(x,t)(u_{eq}(t) + u_{sw}(t))\big]+ s(t)[d(x,t)- \ddot{r}_2] \quad (25)$$

Substituting u_{eq} given in Eq. (23) into Eq. (25) yields:

$$\dot{V}(t)= s(t)[\lambda\dot{e}(t) + f(x,t) + d(x,t)-\ddot{r}(t)]-s(t)g(x,t)\left[\frac{1}{g(x,t)}(\lambda\dot{e}(t) + f(x,t) - \ddot{r}(t))-u_{sw}(t)\right] \quad (26)$$

Eq. (26) can be rewritten as:

$$\dot{V}(t)= s(t)[\, g(x,t)u_{sw}(t) + d(x,t)] \quad (27)$$

To ensure the stability of the system the switching control law can be selected as:

$$u_{sw}(t)= -\frac{1}{g(x,t)}\,\beta\,\mathrm{sign}(s(t)) \quad (28)$$

Substituting u_{sw} given in Eq. (28) into Eq. (27) ensures $\dot{V}(t) \le 0$ when $\beta \ge d_{max} \ge |d(x,t)|$. Hence the stability of the system is guaranteed in terms of the Lyapunov stability theorem [19].

4. TERMINAL SLIDING MODE CONTROL APPROACHES

The TSMC approach to control second order systems was proposed by Ventkaraman and Gulati in 1992 [20]. The TSMC concept is known as a robust control with adjustable finite-time convergence.Therefore, it is called as 'terminal' SMC. There are several different design concepts of terminal sliding modes such as basic TSMC, fast TSMC, nonsingular TSMC, integral TSMC, higher order TSMC, and nested hierarchical TSMC [12].

In the following, the basic TSMC and the FTSMC approaches are outlined.

4.1 BASIC TERMINAL SLIDING MODE CONTROL

Considering the DC motor model presented in Eq. (13), the sliding surface to design the TSM controller can be defined as follows [12]:

$$s= x_2+ \beta|x_1|^{\gamma}\mathrm{sgn}(x_1) \quad (29)$$

where, $\beta>0$ and $0 < \gamma < 1$. Therefore, γ can be defined as $\gamma=q/p$, where p and q are positive odd numbers. In order to guarantee the stability of the controller the TSMC rule can be writtenas [21]:

$$u(t)= - g^{-1}(x,t)\Big((f(x,t)+ \beta\gamma|x_1|^{\gamma-1}x_2+ \eta\,\mathrm{sgn}\,(s)\Big) \quad (30)$$

where $\eta \ge d_{max} \ge |d(x,t)|$.

To analyze the stability of the designed TSMC law, a Lyapunov function may be chosen as:

$$V(t) = \frac{1}{2}s^2(t) \quad (31)$$

Differentiating the Lyapunov function and rearranging it using the system equations introduced in Eq. (13) yield

$$\dot{V}(t)=s[\dot{x}_2+ \beta\gamma {x_1}^{\gamma-1}\dot{x}_1] = s[f(x,t)+g(x,t)u(t)+d(x,t) + \beta\gamma {x_1}^{\gamma-1}\dot{x}_1] \quad (32)$$

Substituting u(t)given in Eq. (30) into Eq. (32) yields

$$=s[f(x,t)+ d(x,t)- f(x,t)- \beta\gamma x_1^{\gamma-1}\dot{x}_1- \eta sgn(s)+ \beta\gamma x_1^{\gamma-1}\dot{x}_1] \quad (33)$$

$$=sd(x,t)-\eta|s|\leq -\eta|s| \quad (34)$$

Since $\eta \geq d_{max} \geq |d(x,t)|$, $\dot{V}(t)\leq 0$ and the stability of the system is guaranteed in terms of the Lyapunov stability theorem [21].

4.2 FAST TERMINAL SLIDING MODE CONTROL

The FTSMC approach is proposed to increase the convergence rate at the basic TSMC method [12,22]. Considering a second order system such as in Eq. (13), the sliding surface to design an FTSM controller can be written as [21]:

$$s_1= \dot{s}_0+\alpha_0 s_0+\beta_0 s_0^{q_0/p_0} \quad (35)$$

where, α_0, $\beta_0>0$, $s_0 = x_1$, and p_0 and $q_0$are positive odd numbers where $q_0<p_0$.
To guarantee the stability of the system the FTSMC law may be designed as [21]:

$$u(t)= - g^{-1}(x,t)\left((f(x,t)+ a_0\dot{s}_0+ \beta_0 \frac{d}{dt} s_0^{q_0/p_0}+\phi s_1+\sigma s_1^{q/p}\right) \quad (36)$$

where $\sigma>0$.

To analyze the stability, a candidate Lyapunov function can be selected as

$$V(t) = \frac{1}{2} s_1^2(t) \quad (37)$$

Differentiating the Lyapunov function and rearranging it using the system equations introduced in Eq. (13) yield

$$\dot{V}(t)= s_1[\ddot{s}_0+\alpha_0\dot{s}_0+\beta_0 \frac{d}{dt} s_0^{q_0/p_0}]=s_1[f(x,t)+g(x,t)u(t)+d(t)+\alpha_0\dot{s}_0+\beta_0 \frac{d}{dt} s_0^{q_0/p_0}] \quad (38)$$

Substituting u(t)given in Eq. (36) into Eq. (38) and rearranging it yields [21].

$$\dot{V}(t)= - \phi s_1^2-\sigma s_1^{(q+p)/p}+s_1 d(t) \quad (39)$$

Since p+q is an even number $-\sigma s_1^{(q+p)/p}+s_1 d(t)$will be satisfied where $\sigma \geq \left|\frac{1}{s_1^{q/p}}\right| |d(t)|$ [20]. Therefore, $\dot{V}(t)\leq 0$ and the stability of the system is guaranteed in terms of the Lyapunov stability theorem [21].

5. SIMULATION RESULTS

In this test, the DC motor model in Eq. (13) is simulated and the fast terminal sliding mode controller presented in Eq. (36) compared with the basic terminal sliding mode controller in Eq. (30) and the conventional sliding mode controller in Eq. (22). The DC motor parameter used in this simulation is presented in Table 1. The SMC parameters are selected as $\beta=2\times10^9$,$\lambda=100$; the basic TSMC parameters are selected as $\eta=8\times10^7$,$\beta=30$, p=9, andq=5; and the FTSMC parameters are selected as $a_0=100$, $\beta_0=15$, p=9, q=5, $p_0=9$, $q_0=5$,$\phi=1$,$\sigma=8\times10^8$. The parameters of all the controllers are selected by the trial-error method. The reference signal is considered as r(t)=1600 in all simulations, also the simulation time is selected as 2s. The control signals created by the conventional SMC, basic TSMC, and FTSMC methods are shown in Figures 2-4, respectively.

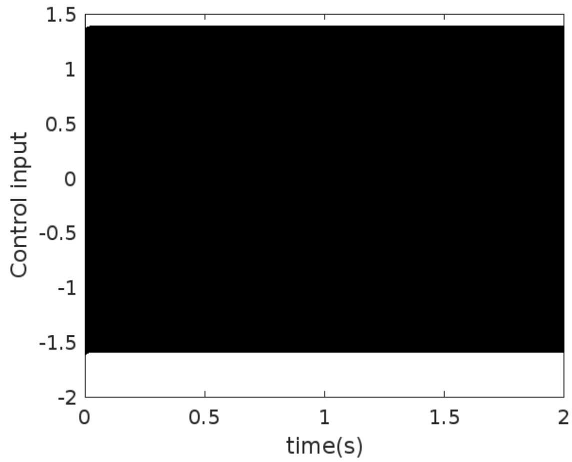

Figure 2: The control signal u(t) using the SMC

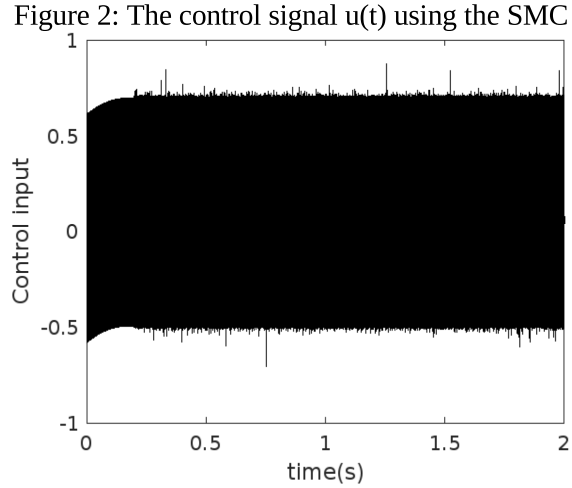

Figure 3: The control signal u(t) using the basic TSMC

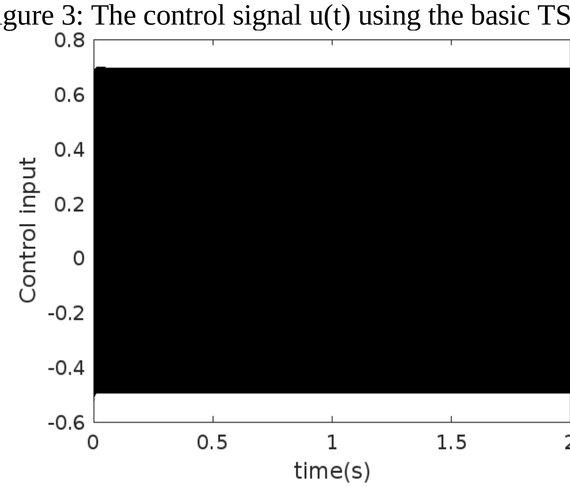

Figure 4: The control signal u(t) using the basic FTSMC

The change in the DC motor speed during the simulations is shown in Figure 5. Also, the performance results are summarized in Table 2 for all three controllers.

Table 2 Performance Results

	Settling Time(ms)	**Overshoot(%)**	**u(t)(v ~range)**
SMC	21.125	0.987	-1.6 –1.4
TSMC	86.243	0.98	-0.5 –0.7
FTSMC	20.965	0.907	-0.5 –0.7

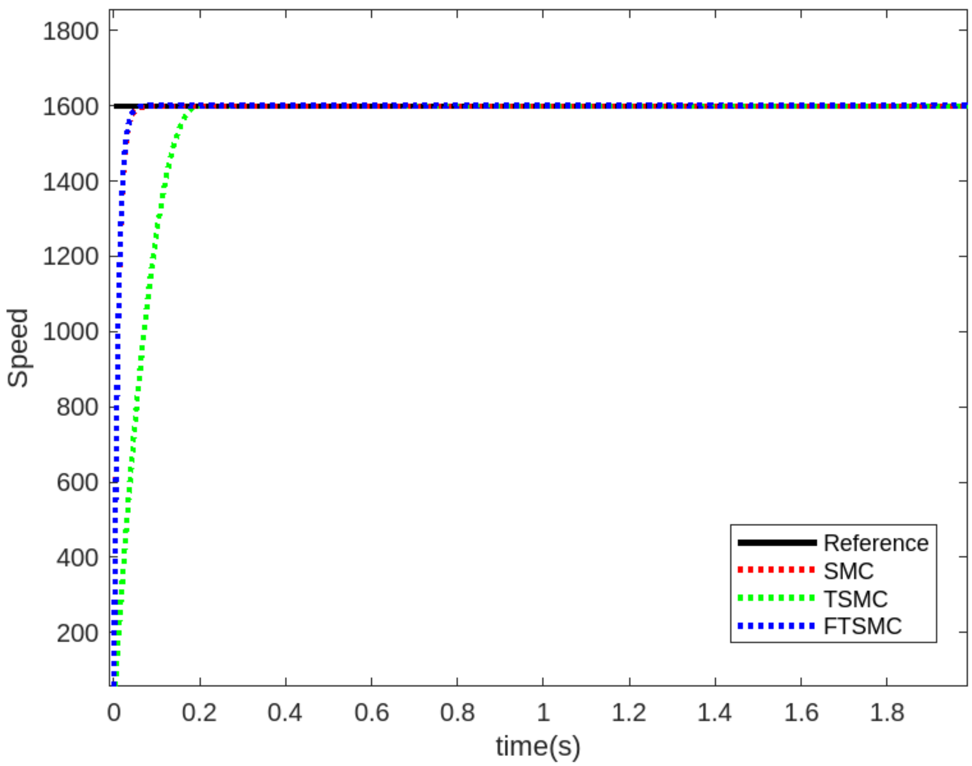

Figure 5: DC motor speed

As shown in Figure 5, all three controllers manage to bring the DC motor speed to the desired level. Both the TSMC and FTSMC methods manage to reduce the chattering compared to the conventional SMC method as shown in Figures 2-4. Also, the settling time FTSMC method is slightly better than the SMC method as presented in Table 2.

6. CONCLUSION

In this study, a fast terminal sliding mode controller is designed to control the speed of a DC motor system. The performance of the designed fast terminal sliding mode controller is investigated through the simulation tests.

The simulation results show that the basic TSMC reduces the total control effort over the conventional SMC with the cost of lack of performance. However, the FTSMC method both results in better performance over the conventional SMC and also reduces the total control effort.

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